1060 Day 25
Fall 2013

5.3 - Logarithmic Functions Part II.

Important things to know quickly about natural logs [and why they're true]

$\ln(1) = 0$ [Because $1 = e^0$]

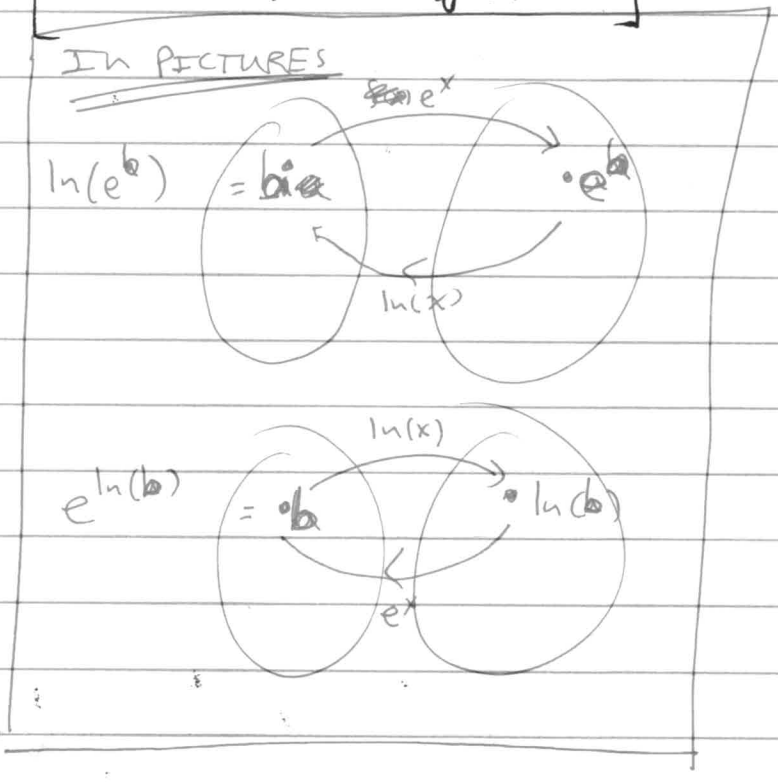
$\ln(e) = 1$ [Because $e = e^1$]

$\ln(e^b) = b$

$e^{\ln(b)} = b$

[because $\ln(x)$ and e^x are inverse functions]

In PICTURES



Solving Equations with exponentials & logarithms

If $e^x = 5$

we can take ln of both sides

$$\ln(e^x) = \ln(5)$$

By the inverse properties, the left side equals x

so $x = \ln(5)$

If $3e^{x+1} + 5 = 11$

solve for x

need to get the e term alone

$$3e^{x+1} = 6$$

$$e^{x+1} = 2$$

Now take ln of both sides

$$\ln(e^{x+1}) = \ln(2)$$

$$x+1 = \ln(2)$$

$$x = \ln(2) - 1$$

To solve equations with ln take e to each side

(if $\log_2$ then take 2 to each side)

Eg: $\ln(5x) = 2$

$e^{\ln(5x)} = e^2$

$5x = e^2$

$x = \frac{e^2}{5}$

Eg: $\log_3(x+5) = 2$

$3^{\log_3(x+5)} = 3^2$

$x+5 = 3^2$

$x = 3^2 - 5$

$x = 4$

Working with $\ln(x)$ is nice because of the...

Laws of Logarithms

$$r \cdot \ln(a) = \ln(a^r)$$

we've seen this one before!

$$\ln(a) + \ln(b) = \ln(a \cdot b)$$

$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

Eg:
$\ln(2) + \ln(3) = \ln(6)$
$\ln(2) - \ln(3) = \ln\left(\frac{2}{3}\right)$
$2 \cdot \ln(3) = \ln(3^2) = \ln(9)$

Eg: Rewrite the expression as a single logarithm

$$\ln(x+2) + \ln(x-1)$$

law 2

$$= \ln\left((x+2)(x-1)\right)$$

$$= \ln(x^2 + x - 2)$$

Eg: $\ln(x+1) + \ln(x-1) - \ln(x+2)$ — law #2

$= \ln\left((x+1)(x-1)\right) - \ln(x+2)$ — law #3

$= \ln\left(\dfrac{(x+1)(x-1)}{x+2}\right)$

$= \ln\left(\dfrac{x^2-1}{x+2}\right)$

Eg: $2 \cdot \ln(x+1) - \ln(x-1)$ — law 1

MUST bring 2 inside FIRST

$= \ln\left((x+1)^2\right) - \ln(x-1)$

$= \ln\left(\dfrac{(x+1)^2}{(x-1)}\right)$

Eg: $2\ln(x) + \frac{1}{3}\ln(x-1)$

law 1 twice

$= \ln(x^2) + \ln\left((x-1)^{\frac{1}{3}}\right)$

$= \ln\left((x^2)\cdot(x-1)^{\frac{1}{3}}\right)$

$= \ln\left(x^2\sqrt[3]{x-1}\right)$

Using These Tools
we can now solve logarithmic equations
like

$$\ln(x) - \ln(x-1) = \ln(2)$$

$$\ln\left(\frac{x}{x-1}\right) = \ln(2)$$

$$e^{\ln\left(\frac{x}{x-1}\right)} = e^{\ln(2)}$$

$$(x-1)\,\frac{x}{x-1} = 2\,(x-1)$$

$$x = 2(x-1)$$

$$x = 2x - 2$$

$$\boxed{2 = x}$$

Solve for x

$$200 = 100 \cdot e^{5x}$$

2 options

ln right away

$$\ln(200) = \ln(100 \cdot e^{5x})$$

$$\ln(200) = \ln(100) + \ln(e^{5x})$$

$$\ln(200) = \ln(100) + 5x$$

$$5x = \ln(200) - \ln(100) = \ln\left(\frac{200}{100}\right)$$

$$x = \frac{\ln(2)}{5}$$

get e^{5x} alone first

$$2 = \frac{200}{100} = e^{5x}$$

$$\ln(2) = \ln(e^{5x})$$

$$\ln(2) = 5x$$

$$x = \frac{\ln(2)}{5}$$

↑ this is shorter & cleaner!

Solve for x

$$200 = x \cdot e^{5}$$

$$x = \frac{200}{e^{5}}$$